

## **General Disclaimer**

### **One or more of the Following Statements may affect this Document**

- This document has been reproduced from the best copy furnished by the organizational source. It is being released in the interest of making available as much information as possible.
- This document may contain data, which exceeds the sheet parameters. It was furnished in this condition by the organizational source and is the best copy available.
- This document may contain tone-on-tone or color graphs, charts and/or pictures, which have been reproduced in black and white.
- This document is paginated as submitted by the original source.
- Portions of this document are not fully legible due to the historical nature of some of the material. However, it is the best reproduction available from the original submission.

CRITERIA FOR THE SELECTION OF FOCUSING ULTRASONIC PROBES

U. Schlengermann

(NASA-TM-75309) CRITERIA FOR THE SELECTION  
OF FOCUSING ULTRASONIC PROBES (National  
Aeronautics and Space Administration) 18 p  
HC A02/MF A01 CSCI 20A

N78-24902

Unclass

G3/71 21226

Translation of "Kriterien zur Auswahl fokussierender Ultraschall-  
Prüfköpfe," Materialprüfung, Vol. 19, Oct. 1977, pp. 416-420.



NATIONAL AERONAUTICS AND SPACE ADMINISTRATION  
WASHINGTON, D.C. 20546 MAY 1978

1. Report No. NASA TM 75309	2. Government Accession No.	3. Recipient's Catalog No.	
4. Title and Subtitle CRITERIA FOR THE SELECTION OF FOCUSING ULTRASONIC PROBES		5. Report Date May 1978	
		6. Performing Organization Code	
7. Author(s) U. Schlengermann		8. Performing Organization Report No.	
		10. Work Unit No.	
9. Performing Organization Name and Address Leo Kanner Associates Redwood City, California 94063		11. Contract or Grant No. NASw-2790	
		13. Type of Report and Period Covered  Translation	
12. Sponsoring Agency Name and Address National Aeronautics and Space Adminis- tration, Washington, D.C. 20546		14. Sponsoring Agency Code	
15. Supplementary Notes  Translation of "Kriterien zur Auswahl fokussierender Ultra- schall-Prüfköpfe," Materialprüfung, Vol. 19, Oct. 1977, pp. 416-420			
16. Abstract  The principles of operation employed in the focusing of a sound field are considered, taking into account the use of solid and liquid coupling media. The focusing limits for a given transducer are investigated. As a diffraction phenomenon, focusing is a function of the system dimensions, the frequency, and the sound velocity. The frequency and the material used for the lenses are in most cases determined in accordance with considerations regarding sound propagation. Changes in the focus are therefore effected mainly by the selection of transducer and lens dimensions. The functions of the focusing factor for a normal immersion probe and a direct contact angle probe are represented in graphs. The deviation of the appropriate parametric values for an ultrasonic probe is illustrated with the aid of examples.			
17. Key Words (Selected by Author(s))		18. Distribution Statement  Unclassified-Unlimited	
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 17	22. Price

# CRITERIA FOR THE SELECTION OF FOCUSING ULTRASONIC PROBES

U. Schlengermann

## 1. Introduction

/416\*

Ultrasonic testing is viewed as a system designed to provide information on the state of a material or workpiece. The display, processing and evaluation of the data constitute the last link of the information chain and as such are meaningful only if the data has been obtained in an optimal manner. The goal of any attempt to optimize the testing system is always to modify the system so as to increase both the sensitivity of the test and the reliability of the data. One approach is the use of focused sound fields. They greatly improve the ratio of signal to noise, and the interaction between the sound field and reflector can be more easily judged if a sharply-defined sound beam is used.

This improvement has its limits, however. Because sound focusing is a diffraction phenomenon, there are limits both from above (no focusing) and from below (strongest focusing).

We shall present a quantitative description of these limits, and discuss the question of which focusing probes or focused sound fields are optimally suited for a given testing problem.

## 2. Focusing the Sound Field

The structure of a focused sound field is shown in Fig. 1.

---

\* Numbers in the margin indicate pagination in the foreign text.



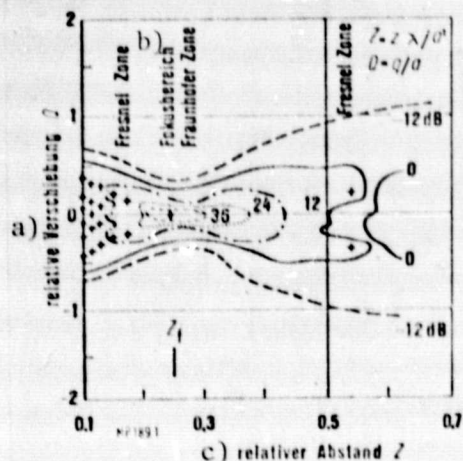


Fig. 1. Measured isobars in a focused sound field for  $Z_f = 0.25$ .

Key: a - Relative displacement; b - Focal region; c - Relative distance  $Z$ .

Here the field is generated by a circular transducer fitted with a spherical lens. The structure shown in Fig. 1 is valid in principle for all focusing systems, however:

- Close to the transducer is a near field (Fresnel zone) which corresponds to the near field of a nonfocusing probe, except that it is shortened by the focusing factor  $Z_f$  [1,2].
- The focus, or point of highest acoustic pressure, is surrounded by a region which corresponds to the entire far field of the nonfocusing probe.

-- At greater distances from the transducer, another Fresnel zone forms which can be viewed as a mirror image of the near field. The stronger the focus (i.e., the smaller the focusing factor  $Z_f$ ), the greater the size of this region [1,2].

For various reasons, the Fresnel zones of the sound field must generally be excluded during testing. Surrounding the focus is the working region of the sound field, called the focal region. It is usually defined as the region with a 6-dB loss of echo from a point reflector relative to the focus.

In material testing, it is of interest to know:

- the longitudinal and transverse dimensions of this region with respect to the acoustic axis, and how these quantities vary

with the parameters of the transducer;

- which sound source produces a focal region suitable for testing a given zone in the material.

/417

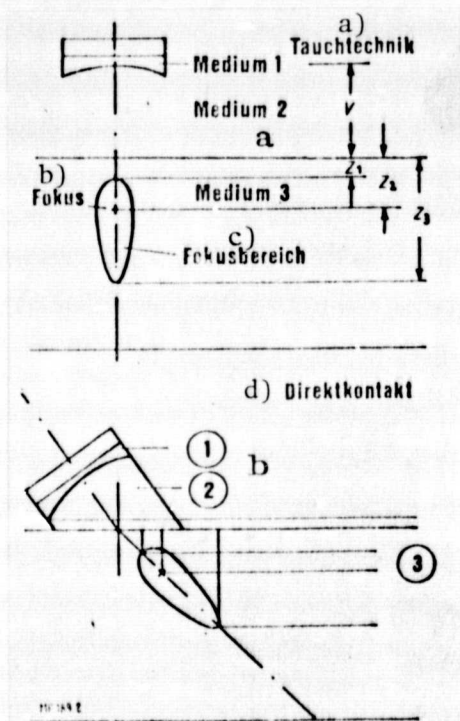


Fig. 2. Focusing with liquid and solid coupling paths.

The shape and position of the focal region (working region) are shown in Fig. 2 for two techniques of sound transmission:

Fig. 2a shows the probe for the immersion technique (here a normal probe), and Fig. 2b a probe for the direct contact method (here an angle probe). Three solid media are employed in direct contact testing, while medium 2 is a liquid in the immersion technique.

The advantage of the immersion method can be readily seen in Fig. 2: By varying the length of the coupling path  $v$  between lens and workpiece, the position of the focal region in the workpiece (medium 3) can be varied as desired within certain limits. The direct contact probe, however, can test only that zone for which it was designed.

### 3. Focusing Limits for a Given Transducer

These limits are best discussed in terms of the change in the focusing factor  $Z_f$  and normalized values [2].

ORIGINAL PAGE IS  
OF POOR QUALITY

The distances are normalized as

$$Z = z\lambda/a^2 \quad R = r\lambda/a^2. \quad (1)$$

The half beam width  $b$  is normalized as

$$B = b/a, \quad (2)$$

where  $a$  is the transducer radius,  $r$  is the radius of lens curvature, and  $\lambda$  is the wavelength.

$Z_f$  expresses the degree to which the near field is shortened relative to the unfocused case. It also represents the relative focal distance and describes the beam width  $b$  at focal distance  $z_f$ :

$$z_f = Z_f a^2 / \lambda, \quad (3)$$

$$2b = \text{const } a Z_f \quad (4)$$

(for a 6-dB echo loss,  $\text{const} = 0.514$ ),

where

$$0 < Z_f \leq 1 \quad (5)$$

is always true [2].

In the case of the nonfocusing probe, the end of the near field is located at the distance  $z = a^2/\lambda$ ; hence  $Z_f = 1$  for the upper limit in the trivial case where the radius of lens curvature  $r = \infty$ .

As a diffraction phenomenon, sound focusing is dependent on the dimensions of the system, the frequency of the beam, and the

sound velocity [1,2]:

$$Z_f = f(a, r, f, c_1, c_2, c_3). \quad (6)$$

For reasons of sound propagation, the frequency  $f$  is usually fixed, as are the materials used for the lens and workpiece. Focusing is then a function mainly of the transducer diameter  $2a$  and the radius of lens curvature  $r$ .

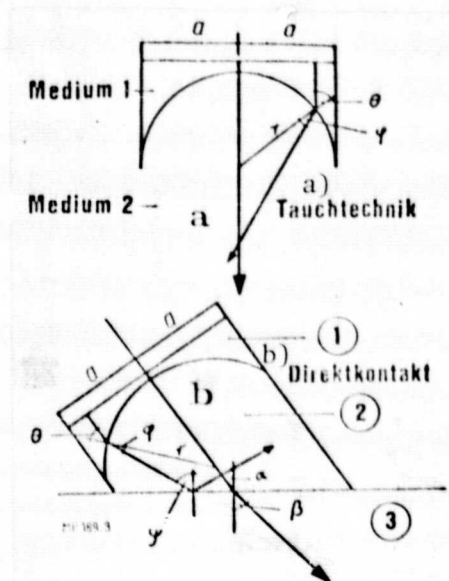


Fig. 3. Focusing limits for the immersion technique and direct contact.

Key: a - Immersion technique; b - Direct contact.

If  $c_1 > c_2$ , as is the case for conventional lenses, all lens curvatures  $r \geq a$  are allowable. The smallest radius is  $r_{\min} = a$  (Fig. 3a). According to eqn. (1), we thus have

$$R_{\min} = \lambda/a \quad (7)$$

for the relative radius  $R$ .

For the desired focusing to be achieved, it is necessary that the entire surface of the transducer emit sound into the workpiece. For conventional immersion probes, this is usually the case if  $r \geq a$ , as Fig. 3 shows. For direct contact probes, however (Fig. 3b), a total reflection takes place on the workpiece surface below certain values of  $r$ , with a consequent indirect decrease in the size of the emitting transducer surface and the occurrence of interfering echos from the lens.

### 3.1. Immersion Technique

If  $c_1 > c_2$ , as is the case for conventional lenses, all lens curvatures



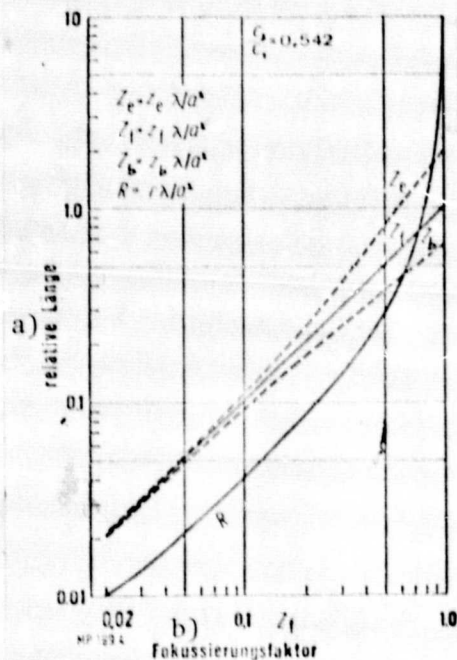


Fig. 4. Functions of the focusing factor  $Z_f$  for an immersion probe.

Key: a - Relative length;  
b - Focusing factor.

probes may be unable to penetrate into the workpiece. This must be prevented. The condition for the minimum allowable radius  $r_{\min}$  is as follows:

$$r_{\min} = a / \sin \theta, \quad (8)$$

where the angle  $\theta$  (Fig. 3b) must satisfy the condition:

$$\theta = \frac{c_2}{c_3} \sin \beta$$

$$\left[ \sqrt{1 - (\sin \theta)^2} \sqrt{1 - \left( \frac{c_2}{c_1} \sin \theta \right)^2} + \frac{c_2}{c_1} (\sin \theta)^2 \right] + \sqrt{1 - \left( \frac{c_2}{c_3} \sin \beta \right)^2} \quad (9) \quad /418$$

$$\left[ \sin \theta \sqrt{1 - \left( \frac{c_2}{c_1} \sin \theta \right)^2} - \frac{c_2}{c_1} \sin \theta \sqrt{1 - (\sin \theta)^2} \right],$$

The corresponding focusing factor  $Z_{f\min}$  can be plotted in a generally valid graph for a constant index of refraction  $c_2/c_1$  for immersion probes, as shown in Fig. 4. The graph also gives the relative distances  $Z_b$  for the beginning of the focal region and  $Z_e$  for the end of this region as a function of the focusing factor  $Z_f$  (calculations after [1]). The width  $2b$  of the sound beam at the focus can also be determined from eqn. (2) with the factor  $Z_f$ . The calculation of the upper and lower limits for the length of the working region and the beam width is illustrated in Example 1 (see Appendix).

### 3.2. Direct Contact Technique

Depending on the media employed (Fig. 3b), the waves from the edge of the transducer in direct contact

where  $\beta$  is the incident beam angle in the workpiece.

After normalization, eqn. (8) leads to

$$R_{\min} = (1/\sin \theta) \cdot (\lambda/a). \quad (10)$$

The minimum focusing factor  $Z_{f\min}$  for this normalized radius  $R_{\min}$  can again be determined graphically.

For direct contact probes, normalization of the graphs is less helpful because the coupling path in the wedge has various relative values depending on the probe and the angle of incidence. In this case it is better to use special graphs like that shown in Fig. 5 for a 2-MHz transverse-wave probe with  $2a = 50$  mm and  $\beta = 45^\circ$ . This graph was calculated using programs of the BAM [3] for the calculation of direct contact probes. Due to the oblique incidence of the beam, the lens must have different curvatures perpendicular ( $\perp$ ) and parallel ( $\parallel$ ) to the plane of incidence so that a beam of approximately circular cross-section will be produced in the focal region [3]. The beginning ( $Z_b$ ) and end ( $Z_e$ ) of the focal region in the steel are indicated in the graph as projected distances (Fig. 2b), thus making it easier to determine the depth zone tested. The lower focusing limit  $Z_{f\min}$  according to eqn. (8)-(10) is also shown.

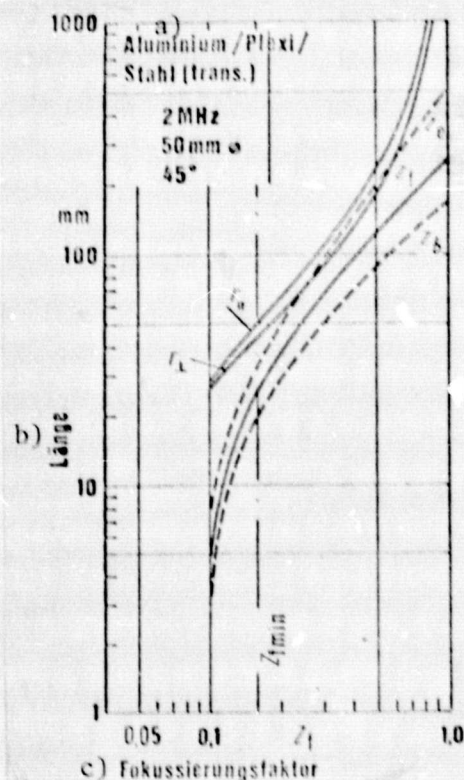


Fig. 5. Functions of the focusing factor  $Z_f$  for a direct contact angle probe.

Key: a - Aluminum/Plexi-glas/steel (transverse); b - Length; c - Focusing factor.

The factors  $1/\sin \theta$ , which are important for judging the focusing of direct contact probes, are given in Table 1 for several 2-MHz probes ( $0^\circ$  long. incidence,  $45^\circ$  trans. incidence) currently on the market. The lens material is aluminum/Plexiglas.

Table 1. Important Factors for Judging the Focusing of Direct Contact Probes

a) Schwinger- dmr. 2a mm	b) Einschall- winkel $\beta^\circ$	$1/\sin \theta$	$r_{\min}$ mm	$Z_{\min}$
34	0	1.40327	23.86	0.167
	45	1.73712	29.53	0.204
50	0	1.40327	35.08	0.116
	45	1.73712	43.43	0.142
75	0	1.40327	52.62	0.078
	45	1.73712	65.14	0.096
100	0	1.40327	70.16	0.059
	45	1.73712	86.86	0.072

Key: a. Transducer diameter 2a, mm  
b. Angle of incidence  $\beta^\circ$   
(Commas in tabulated material are equivalent to decimal points.)

The focusing limits of a direct contact probe are determined in Example 2 (see Appendix).

#### 4. Matching the Focus to the Testing Problem

The graphs in Fig. 4 and 5 for immersion and contact probes make it possible to derive the parameters of the focal region or determine the focusing limits for any probe if the quantities  $a$ ,  $r$ ,  $f$ ,  $c_1$ ,  $c_2$ ,  $c_3$  are known, eqn. (3), (4) and (6). The problem is more difficult if a suitable focusing probe must be found for a given workpiece, i.e. for a given zone in the object under test or given reflector dimensions.

Often a probe is sought which will just cover a given test



zone, in which case secondary conditions are usually made for the coupling distance  $v$  or for the transducer diameter  $2a$  or near field length  $a^2/\lambda$ . The zone is covered by the focal region if

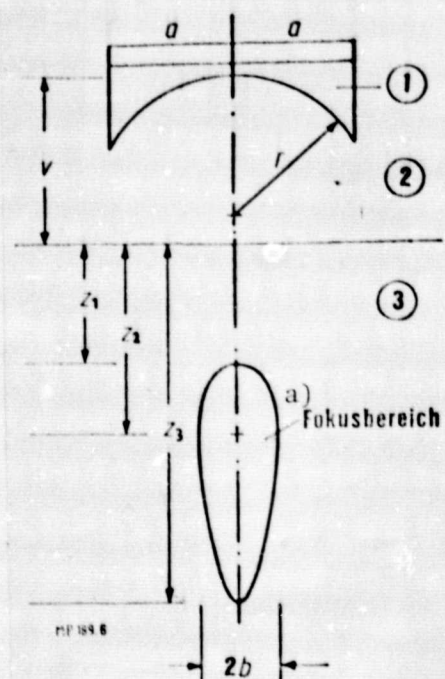


Fig. 6. Matching the focusing to the test problem.

Key: a - Focal region.

the beginning of the region  $Z_b$  coincides with the beginning of the test zone at distance  $z_1$ , and the end of the focal region  $Z_e$  is at  $z_3$  (Fig. 6). The parameters on which the value of  $v$  or  $a^2/\lambda$  should depend are expressed in generalized form as  $p$  and  $q$ . We thus obtain the following equation system:

$$Z_a N_2 - r = (c_3/c_2) z_1, \quad (11)$$

$$Z_e N_2 - r = (c_3/c_2) z_3, \quad (12)$$

$$N_2 = f(p), \quad (13)$$

$$r = g(q). \quad (14)$$

This system of four equations can be solved for the two unknowns  $N$  and  $r$  only if eqn. (15) and (16) are both satisfied:

$$(c_3/c_2)(z_3 - z_1) = f(p)(Z_e - Z_a), \quad (15)$$

$$(c_3/c_2)(\frac{1}{2} Z_a - z_1 Z_e) = g(q)(Z_e - Z_a). \quad (16)$$

All quantities necessary for using the focused probe can be derived from these solutions. All the quantities associated with the transducer diameter  $2a$  can be derived from eqn. (15), and the coupling distance  $v$  from eqn. (16).



#### 4.1. The Prevention of Interfering Multiple Echos from the Coupling Path

This secondary condition is made in almost every immersion testing problem:

$$v > (c_2 / c_3) z_3 .$$

Thus, there should be no interfering echos from the coupling path over the testing region from  $z_1$  to  $z_3$ . With eqn. (14) and (17), we obtain the solution:

$$\frac{Z_e}{Z_a} < \frac{z_3 [1 + (c_2/c_3)^2]}{z_1 + (c_2/c_3)^2 z_3} . \quad (18)$$

Thus we see that only those factors  $Z_f$  are allowed for which  $Z_e/Z_b$  satisfies the condition of eqn. (18). Since the function  $Z_e/Z_b$  is limited, as Fig. 7 shows, there will be no solution to the problem in certain cases. The necessary transducer radius  $a$  is obtained from eqn. (15) with eqn. (13).

$$N_2 = a^2 / \lambda_2 = \frac{c_3}{c_2} \left( \frac{z_3 - z_1}{Z_e - Z_a} \right) . \quad (19)$$

With eqn. (16) and (14), we obtain the corresponding coupling distance  $v$  as:

$$v = \frac{c_3}{c_2} \left( \frac{z_3 Z_a - z_1 Z_e}{Z_e - Z_a} \right) . \quad (20)$$

It is desirable in every case to choose the smallest possible transducer size or the shortest possible coupling path, and thus to select a focusing factor  $Z_f$  whose quotient  $Z_e/Z_b$  satisfies eqn. (18) with approximate equality. See Example 3 for illustration (Appendix).

#### 4.2. Necessary Beam Width $2b$

If the sound field must have a certain lateral resolving power, there is a certain maximum beam width  $2b$  at the focus which must not be exceeded. From eqn. (3) and (4), we thus have

$$N_2 = a^2/h_2 = \frac{4b^2}{k^2 Z_f^2 \lambda_2} \quad (21)$$

Fig. 7. Functions of the focusing factor  $Z_f$  for the immersion technique.

Key: a - Focusing factor.

for the fall of echo amplitude, where  $k$  is a constant.

Using eqn. (15) and (13), we obtain the solution

$$\left( \frac{Z_e - Z_b}{Z_f^2} \right) = \frac{k^2 c_3 (z_3 - z_1)}{4b^2 f} \quad (22)$$

(for -6dB,  $k = 0.514$ ).

The function  $(Z_e - Z_b)/Z_f^2$  is also plotted over  $Z_f$  in Fig. 7. This function, too, is limited. The problem at hand can be solved only if a solution to eqn. (22) is possible within this range of values. If there is a solution, it is unique in  $a$  and  $v$ . These

quantities are determined from eqn. (13) through (16) analogously to the case in 4.1. See Example 4 for illustration (Appendix).

#### 4.3. Probe with Minimum Beam Width or Coupling Distance

In section 4.1 it was determined which probes are suitable for covering a specified zone in the workpiece. But it is important to know which of these possible solutions yields the smallest beam width at the focus (and thus the best resolving power) or can be employed with the shortest coupling path possible.

As eqn. (22) shows, the beam width is minimal when the value of  $(Z_e - Z_b)/Z_f^2$  is maximal. However, the coupling distance  $v$  is minimal according to eqn. (16) when  $Z_e/Z_b$  is maximal. Since the latter quantity increases monotonically with  $Z_f$ , the coupling distance can be kept small by employing as weak a focus as possible, or in other words by making  $Z_f$  as large as possible.

The beam width  $2b$  does not increase monotonically with  $Z_f$ , as Fig. 7 shows. The value of  $(Z_e - Z_b)/Z_f^2$  is maximal at a focusing factor of 0.3. If the solution  $Z_f = 0.3$  is allowed by eqn. (18), this probe will yield the smallest possible beam width which will still cover the zone to be tested.

Thus, from all the solutions which satisfy eqn. (18), one can always select one solution which yields either a small beam width (usually with a large coupling distance) or a small coupling distance (usually with a larger beam width). This is illustrated in Example 5 (see Appendix).

#### 5. Summary

As a diffraction phenomenon, acoustic focusing is dependent on the diameter of the transducer relative to the wavelength. Since only the Fraunhofer region is generally employed for



ultrasonic testing, the dimensions that can be achieved for the focal region are a function of the transducer dimensions and the wavelength, or to be more precise, the ratio  $a/\lambda$ . In general, a stronger focus can be achieved with immersion probes than with direct contact probes for the same transducer and frequency.

Matching the focused sound field to the testing problem imposes conditions on the coupling distance  $v$  and transducer size  $2a$  which must always be satisfied simultaneously. Graphs like those in Fig. 4 and 7 are a useful aid in finding the solution.

In testing practice, it would naturally be desirable to work with the smallest transducer dimensions and shortest coupling path possible. Some trade-offs must be made, however, and the weakest focus possible (maximum value of  $Z_f$ ) offers the most favorable solution in practice.



## REFERENCES

1. Schlengergermann, U., "Formation of sound field by flat ultrasonic sources with focusing lenses," Acustica 30/6, 291-300 (1974).
2. Frielinghaus, R., "Discussions on ultrasonic testing with focusing probes," Meeting of the DGZfP on New Techniques for the Analysis of Ultrasonic Test Data, Feb. 24, 1977, Berlin.
3. Wüstenberg, H., Kurzner, J. and Möhrle, W., "Focusing probes as a means of improving error estimation in the ultrasonic testing of thick-walled reactor components," Materialprüf. 18/5, 152-61 (1976).

## APPENDIX

### Example 1

Given:

$f = 5.00$  MHz,  $2a = 20.00$  mm,  $c_1 = 2.73$  km/s (Plexiglas),  $c_2 = 1.48$  km/s (water);

Without focusing:

$Z_{fmax} = 1.000$ ,  $Z_{emax} = 2.000$ ,  $Z_{bmax} = 0.670$  (Fig. 4),  $z_{amax} = 676.00$  mm,  $z_{bmax} = 226.00$  mm eqn. (1),  $(z_e - z_b)_{max} = 450.00$  mm,  $2b_{max} = 5.14$  mm eqn. (4);

Strongest focusing:

$R_{min} = 0.0296$  eqn. (7),  $Z_{fmin} = 0.0712$ ,  $Z_{emin} = 0.0763$ ,  $Z_{bmin} = 0.0669$  (Fig. 4),  $z_{emin} = 25.78$  mm,  $z_{bmin} = 22.60$  mm eqn. (1),  $(z_e - z_b)_{min} = 3.18$  mm,  $2b_{min} = 0.370$  mm eqn. (4).

### Example 2

Given:

$f = 2.00$  MHz,  $2a = 50.00$  mm,  $\beta = 45.00^\circ$ ,  $c_1 = 6.32$  km/s (aluminum),  $c_2 = 2.73$  km/s (Plexiglas),  $c_3 = 3.25$  km/s (steel, trans.);

Without focusing:

$Z_{fmax} = 1.0000$ ;

Projected depth level:

$z_{emax} = 516.00$  mm,  $z_{bmax} = 168.00$  mm, steel, Fig. 5,  $(z_e - z_b)_{max} = 348.00$  mm,  $2b_{max} = 12.84$  mm, eqn. (4);

Strongest focusing:

$R_{min} = 0.0948$  eqn. (10),  $r_{min} = 43.43$  mm eqn. (1),  $Z_{fmin} = 0.1420$  Fig. 5;

Projected depth level:

$z_{emin} = 25.73$  mm,  $z_{bmin} = 15.40$  mm steel, Fig. 5,  $(z_e - z_b)_{min} = 10.33$  mm,  $2b_{min} = 1.824$  mm eqn. (4).

### Example 3

Given:

$f = 2.00$  MHz,  $c_1 = 2.73$  km/s (Plexiglas),  $c_2 = 1.48$  km/s (water),  
 $c_3 = 5.92$  km/s (steel, long.),  $z_1 = 50.00$  mm,  $z_3 = 80.00$  steel;

Calculated:

$Z_e/Z_b < 1.5455$  eqn. (18),  $Z_f < 0.2547$  Fig. 7;

Selected:

$Z_f = 0.200 < 0.2547$ ,  $Z_e = 0.244$ ,  $Z_b = 0.171$  Fig. 4,  $a^2/\lambda_2 =$   
1644.00 mm eqn. (15) and (13),  $a = 34.88$  mm,  $2a \approx 70$  mm,  $v =$   
81 mm eqn. (16) and (14),  $2b = 3.58$  mm eqn. (4).

### Example 4

Given:

$f = 2.00$  MHz,  $c_1 = 2.73$  km/s (Plexiglas),  $c_2 = 1.48$  km/s (water),  
 $c_3 = 5.92$  km/s (steel, long.),  $z_1 = 50.00$  mm,  $z_3 = 80.00$  steel;

Required:

$2b = 3.64$  mm;

Calculated:

$Z_e - Z_b/Z_f^2 = 1.771$  eqn. (22),  $Z_{f1} = 0.1908$ ,  $Z_{f2} = 0.3606$  Fig. 7.

Since it is required by Example 3 that  $Z_f < 0.2547$ ,  $Z_{f2}$  is not a solution to the present problem. If the testing zone is covered, it is true that the required beam width  $2b$  is present. But the coupling distance  $v$  needed for complete coverage is so short that multiple interfering echos arise.

$Z_{e1} = 0.2306$ ,  $Z_{b1} = 0.1644$  Fig. 4,  $a^2/\lambda_2 = 1813.00$  mm eqn. (15) and (13),  $a = 36.62$  mm,  $2a \approx 74$  mm,  $v = 98.00$  mm eqn. (16) and (14),  $2b = 3.59$  mm eqn. (4).

### Example 5

Given:

$= 2.00$  MHz,  $c_1 = 2.73$  km/s (Plexiglas),  $c_2 = 1.48$  km/s (water),  
 $c_3 = 5.92$  km/s (steel, long.),  $z_1 = 50.00$  mm,  $z_3 = 110.00$  mm  
steel;

Calculated:

$Z_e/Z_b < 2.0549$  eqn. (18),  $Z_f < 0.4566$  Fig. 7;

Selected:

$Z_{f_1} = 0.4500 < 0.4566$ ,  $Z_{f_2} = 0.3000 < 0.4566$ ;

Calculated:

$z_{e_1} = 0.6935$ ,  $z_{b_1} = 0.3401$ ,  $z_{e_2} = 0.4031$ ,  $z_{b_2} = 0.2428$ , Fig. 4;  
 $a_2^2/\lambda_2 = 679.0000$  mm,  $a_2^2/\lambda_2 = 1497.0000$  mm, eqn. (15) and (13),  
 $a_1 = 22.42$  mm,  $a_2 = 33.29$  mm,  $2a_1 \approx 45$  mm,  $2a_2 \approx 67$  mm,  $v_1 = 31.00$   
mm,  $v_2 = 164.00$  mm, eqn. (16) and (14),  $2b_1 = 5.19$  mm,  $2b_2 =$   
5.13 mm, eqn. (4).